Generating Discrete Events

Wednesday, March 05, 2014 12:51 PM

Last time . Generating continuous RVs

Today : Generating discrete events

↓

Discrete Event Simulations

Poisson Process (PP)

There are two types of this

- HPP : constant λ (homogeneous)
- NHPP : λ is dependent on t (Non-)

Recall that the times between successive events in HPP are independent exponential with parameter λ.

To generate HPP, we simply find the cumulative sum of these exp. RVs to get the time of occurrence for events.

→ Easily generated by $-\frac{1}{\lambda}\ln(U)$, $U \sim \mathcal{U}(0,1)$

Extension of HPP concept

① NHPP

steps for generating NHPP

i) Find λ such that $\lambda(t) \leq \lambda$ for all $t \leq T$ (end time)

ii) Generate HPP with rate λ

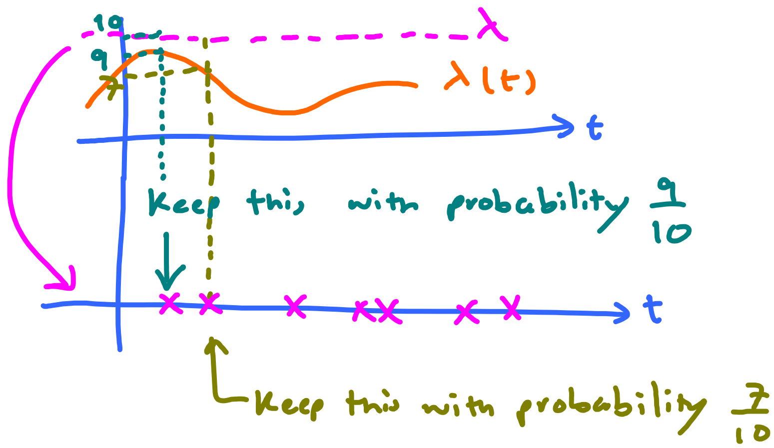
iii) Keep the event that occurs at time t with

- Thinning
- random

- random sampling

iii) Keep the event that occurs at time t with probability $\frac{\lambda(t)}{\lambda}$.

Ex.



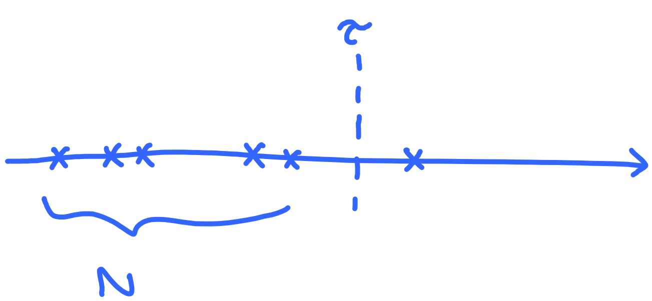
② Renewal Processes

exponential → another pdf

③ Poisson RV

Recall that if we count the # events that occur in a time interval of length τ in a Poisson process with rate λ,

then this number $\sim \mathcal{P}(\underbrace{\lambda\tau}_{\alpha})$.



$\alpha = \lambda\tau$

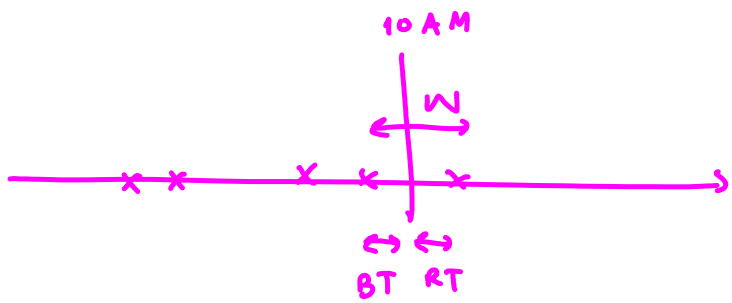
$$N = \max\left\{ n : \sum_{i=1}^{n} -\cancel{\frac{1}{\lambda}} \ln(U_i) \le \overset{\alpha}{\cancel{\overset{\cancel{\lambda}}{\cancel{\tau}}}} \right\} \sim \mathcal{P}(\overset{\alpha}{\cancel{\lambda\tau}})$$

④ Markov chains < discrete time ← Back to Bernoulli trials
- allow more than 2 possibilities

④ Markov chains

- discrete time ← Back to Bernoulli trials
  - allow more than 2 possibilities
  - allow dependency among trials
- limit ↓
- continuous time

Waiting-time paradox



You are more likely to experience longer interval.

(which implies longer time to wait than you might expect.)

Suppose the time btw adjacent bus arrivals is $\sim \mathcal{E}(\lambda)$.
Then, the average time $= \frac{1}{\lambda}$.

Turn out that when you consider the interval that you actually falls into, the average length of it is $\frac{2}{\lambda}$.
This is because you are more likely to fall into longer intervals.